Dear reviewers,

Thank you for your time to review our paper. We truly appreciate all your helpful comments, suggestions, and corrections for improving this paper. In response to your suggestions, we have outlined the changes and provided the necessary explanations.

In response to the comments of Reviewer #1:

Abstract

-- P1 L19: I am not sure why you use italics in various spots in the abstract

**In this revision, we only use italics on the newly defined terms and phrases like "G-zombie" or "G-move".**

-- P1 L24 and throughout the paper: I would use “the survivor” instead of “survivor”

**We made this change for most of “survivor”s throughout the paper.**

Introduction

-- P1 L47: “cops should not” should be “cops need not”

-- P1 L48: “They can either hold...” should be its own sentence.

**We applied these changes in this revision.**

-- P2 L15: There should not be a comma after “Capture time in a game”

-- P2 L17: “exists” should be “exist”

**We corrected these in this revision and moved capture time related definitions from section 3 to here.**

-- P2 L26 and throughout the paper: Notation for Cartesian product looks strange. Is there a better way to typeset?

**We shortened the square notation.**

-- P2 L41, L45, L47: Instead of 3, 4, 5, write “Section 3”, “Section 4”, “Section 5”

**We have changed these in this revision.**

Zombie number of the Cartesian product of two graphs

-- P3 L6: “notations” should be “notation”

-- P3 L8: It would be clearer to write “For 1 ≤ i ≤ m, define Gi to be the subgraph induced by..”

-- P3 L9: It would be clearer to write “Similarly, for 1 ≤ j ≤ n, define Hj to be the subgraph induced by..”

-- P3 L9: It is redundant to define (j, i) as the common vertex Gi and Hj . You could remind the reader of the notation.

-- P3 L20: It would be clearer to write ”Define distI (j, k) to be the distance between vertices j and k on a graph I. Define the length of a path P to be len(P).”

-- P3 L23: It would be clearer to write “For a vertex (u,v) ∈ G􏰀H, define its G-equivalent vertex to be u ∈ V (G). You should also make the definition of G-equivalent graph more formal then “where we put”.

**We have changed these in this revision.**

-- P3 L52: It would seem clearer to talk about G- and H-edges rather than moves.

-- P4 L6: I think this proof should be more formal. Instead of writing some moves are happening, you could specify that the path of the same length with G-edges before H-edges has all G-edges in Gy and all H edges in Hu.

**Since Reviewer #3 suggested to remove the proof of this lemma and only state it as a simple fact, we have done so and this proof is not included in the new revision of the paper.**

-- P4 L30: I think the sentence “If a = y and the survivor makes an H-move, ...” is not necessary (the zombie strategy does depends only on the position when it is their turn).

**We have removed this sentence.**

-- P4 L39: I think the sentence “Now for each G-move...” is not clear. Is the point you’re trying to make that whenever the survivor makes an H-move, that the zombies will all be on Hx?

**Yes. Instead of H-equivalent we should have used H\_x. We corrected this in the new revision.**

-- P4 L49, 53: It would be clearer to write “Theorem 2” rather than “theorem proved above” and “proved conjecture”

**We have changed these in the new revision.**

Capture time in Cartesian product of graphs

-- P5 L23 and throughout: I think in Theorem 5 you could replace diameter of the graph with radius (just start the cops at the vertices that minimize the length of the longest path). Also you could subtract 1 from the capture time since it is only necessary for the G or H-zombies to catch the survivor, not both.

**We applied these comments in this revision.**

-- P5 L26: Theorem 2 should be captitalized.

-- P5 L35: “total number” should be “the total number”

**We have changed these in this revision.**

Limited capture time zombie number problem is NP-Hard

-- P5 L42: Quotation mark is backward

-- P5 L46: You should include a reference that dominating set is NP-Hard.

-- P5 L54: “Also NG[u] represents...” should be “Let NG[u] represent...” and you should specify whether this set includes u itself (is it the closed or open neighborhood)?

-- P6 L6: First sentence should be “The LCZk problem...”

-- P6 L7: The instance should be: “Let G = (V,E) be a simple undirected graph and let zc and k be positive integers.”

-- P6 L9: The second sentence of the question is not needed (or should be placed outside the formal definition).

-- P6 L13: The instance should be “Let G = (V,E) be a simple undirected graph and let d be a positive integer.”

**We applied all these corrections in this revision.**

-- P6 L30: You should explicitly state adjacencies for (v, i)’s.

**We stated the exact path in the new version. We think it is much clearer now and adjacencies are stated as well.**

-- P6 L43: Why are the k’s capitalized in the figure?

**It was a mistake. We used lowercase letters in this revision.**

-- P6 L48: Also note the number of edges in Gk is O(n).

**Reviewer #3 noted that this line is too obvious and it is not needed to be mentioned. In this revision we just stated that Gk can be built in polynomial time.**

-- P6 L50: It would be clearer to say: “Let S be the set of vertices v ∈ V (G) such that v ∈ S′ or (v, i) ∈ S`.”

**We changed this in the new revision.**

-- P7 L17: I am not convinced by the proof of Lemma 7. It is certainly true that if the survivor moves n + 1 times, they will be on the same vertex twice. But that does not imply that the zombies will be in the same position both times (even if their total distance is the same). Without this fact I don’t think you can argue that the survivor could repeat the same moves.

**The proof of Lemma 7 was wrong and it is removed from this revision. Since it was not used anywhere else in the paper, other theorems still hold.**

Zombie number problem is NP-Hard

-- P7 L45: The instance should be: “Let G = (V, E) be a simple undirected graph and let zc be a positive integer.”

-- P8: Proof of Theorem 8 is correct, but could be a bit clearer.

**We changed these in the new revision and rewrote the proof of Theorem 8 (Theorem 7 in this revision) to make it clearer.**

In response to the comments of Reviewer #2:

-- P1. Line 29. The final G in the Abstract needs to be in math mode.

-- P1. Lines 43-45. Since the zombie player starts first, why present the survivor's details before the zombie's details?

-- P2. Lines 15-16. The attempt to define capture time has too many grammar mistakes to be understood. Please revise this.

-- P2. Lines 39-47. Don't say "2", "3", etc. Say "Section 2", etc.

-- P3. Line 16. H-Move should be H-move

**We applied all these corrections in this revision.**

-- P3-4. In the proof of Lemma 1 there are several mentions of moves. However, the lemma does not involve any players, and so there are no entities that would be in motion. I believe that several of the references to "moves" in this proof should instead refer to "edges".

**Since Reviewer #3 suggested to remove the proof of this lemma and only state it as a simple observation, we have done so and this proof is not included in the new revision of the paper.**

-- P5. L26. Theorem 2 (not theorem 2)

-- P5. L42. The quotes are in the wrong direction. This also happens elsewhere in the paper, such as in the References.

**We have changed these in this revision.**

-- P5. L44. I believe that the class that has been described is P, not NP. The authors should ensure that NP is correctly defined.

**This part was removed in the new version.**

-- P5. L53. Do the authors truly want the closed neighborhood of u, and not the open neighborhood? If yes, then they should say so. Alternatively, if they want the open neighborhood, then they should not use the notation for closed neighborhood.

**We refer to this as closed neighborhood in the new version.**

-- P6. L52-53. Surely vertex $(u,k-1)$ is adjacent to an other vertex, and so the statement that "there is no vertex at distance $k$ or less" is false.

**We meant there is no zombie at distance k or less. We changed it in this revision.**

-- P7. L22. I believe "each two rounds" is meant to be "each round"

-- P7. L28-30. I agree with the pigeonhole principle statement. But I am not yet convinced that the final sentence of the paragraph is correct. Couldn't the positions of the players have now changed from the previous time when the survivor was on the previously visited vertex, and if so, might there now be an opportunity for distances to be reduced in the coming moves (which will not be the same moves as before)?

**The proof of Lemma 7 was wrong and it is removed from this revision. Since it was not used anywhere else in the paper, other theorems still hold.**

In response to the comments of Reviewer #3:

2. Technical Details

**1: We changed the definition and now refer to this game as "Zombies and Survivor" instead of "Zombies and Survivors"**

**2: We changed this in the new version. Now the zombies player is given "z" zombies instead of choosing "z" himself.**

**3: We changed this in the new version and made a customized notation "z" for z\_c. By c in z\_c we meant "count" but it was definitely not clear!**

**4: A new sentence is added to this revision to emphasize the rule.**

**5: We moved the definition of capture time from page 5 to section 1 and also added the reference you mentioned to clarify this parameter.**

**6: We replaced this with "capt" function in this revision.**

**7: "G-equivalent graph" is a new copy of graph "G", where we put each survivor and zombies, on their "G-equivalent" position. Since it is a graph, we believe that "G-equivalent graph" suits it more than "G-equivalent game position".**

**8: As you suggested, we removed the proof and just stated the lemma.**

**9:**

**(a) Above figure 2, we stated that survivor is located at (x,y). In this revision we have added a new sentence to remind the reader.**

**(b) p\_H refers to a single object which is a path in graph H. Every G-zombie will move along this path in its H-subgraph. Also choosing a single path is key to this proof, since if we use different paths for our zombies, they might end up in different H-subgraphs which makes us unable to discuss the game on a single H.**

**(c) We applied your suggestion in this revision**

**(d) We made some changes to make the proof shorter and easier to understand. Please let us know if it needs to be clearer.**

**10: We removed the appendix and added it to the supplementary materials of the paper.**

**11: We changed this in the new version.**

**12: We added the reference to this revision.**

**13: We corrected the definition and also moved it to above figure 1.**

**14: We replaced diameter with radius and wrote "Let $rad(G)$ represent the $G$'s radius".**

**15: These lines are removed in this revision.**

**16: We changed this to closed neighborhood.**

**17: k is not an input to the problem and is just a parameter. We applied your suggestion and changed the phrase to “INSTANCE of LCZ\_k problem”.**

**18: We changed this in the new version.**

**19: We have applied these changes in this revision.**

**20: We included this reference and also mentioned the cops and robbers version of the problem at page 2.**

**21:** **The proof of Lemma 7 was wrong and it is removed from this revision. Since it was not used anywhere else in the paper, other theorems still hold.**

**22: The reference is included in this revision.**

**23: Thanks for your comment, we applied your suggestion to this revision.**

**24: In this revision we use B\_i instead of K\_i to make the sentence clearer.**

**25: We applied your comment in this revision.**

**26: We now state that the game is played on a “simple connected graph”**

**27: These parts of proof were modified using Reviewer #1's comments.**